

## Ice Load Prediction During Indentation

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### INTRODUCTION

The extraction of hydrocarbon resources from the Arctic in an economical and safe manner poses many technical challenges to offshore engineering. At the root of these problems is the severe environment created by perennial ice features that impart global forces and local pressures on structures that are several times greater than those from waves in non-Arctic environments. Typically, two levels of ice loading are considered for design purposes. Global ice loads govern the overall structural geometry and dimensions as well as the foundation design, while local ice pressures dictate wall thicknesses and local framing, and may well govern structural cost. It is widely recognized that significant uncertainties exist in current ice load prediction models and that some design loads may be over-estimated by an order of magnitude.

Uncertainties in existing ice load models arise primarily from five sources:

- Incomplete knowledge of the mechanical behavior of ice, including temperature and fracture behavior.
- Empiricism in existing theoretical models resulting from the use of approximate analysis methods.
- Inadequate modeling of the contact forces at the ice-structure interface.
- Neglecting the effect of scale/size on material strength.
- Not accounting for the finiteness of environmental and other forces driving the ice features.

Numerical models that simulate ice-structure interaction processes on a computer can be used to quantify these uncertainties and to better predict global and local ice loads. In contrast to analytical methods, such models can realistically simulate the interaction accounting for spatial-temporal variability in the mechanical behavior of ice and for multiple modes of failure in ice.

This paper summarizes the results to date from research underway at MIT which is concerned with the development and application of numerical models for quantifying ice-structure interaction processes. This research involves the following three major areas of study:

1. Development of constitutive models to characterize the mechanical behavior of ice.
2. Development of finite element methods of analysis to account for the simultaneous occurrence of viscous (rate dependent) and fracture behavior in ice, and time varying contact between ice and structure.
3. Numerical simulation of ice-structure interaction processes for selected ice features and structural configurations to predict global forces and local pressures.

In particular, the objective of this paper is to identify and to quantify the major factors affecting ice load prediction during indentation involving interaction of an ice sheet with a rigid cylindrical structure. This is a problem of general interest in the Arctic since it represents an important loading condition both for cylindrical structures and for conical structures with grounded rubble pile or accreted ice foot.

Previous theoretical models for prediction of global forces during indentation are based on approximate methods of analysis. They include: (a) the upper and lower bound, plasticity solutions of Michel and Toussaint (1977) using the von-Mises yield criterion, of Croasdale et al. (1977) using the Tresca criterion, and of Ralston (1978) using a transversely isotropic and pressure sensitive yield function developed by Reinicke and Ralston (1977); (b) the reference stress, power law creep solution of Ponter et al. (1983); and (c) the upper bound, power law creep solutions of Bruen and Vivatrat (1984), Vivatrat, Chen and Bruen (1984), and Ting and Shyam Sunder (1985a) using an isotropic formulation and of Vivatrat and Chen (1985) using an anisotropic formulation. The API Bulletin 2N guidelines contain the results from Ralston's (1978) study based on plasticity theory. The use of plasticity theory, which does not account for time (rate)

dependent material properties, is an empirical definition of ice sheet. The models are not generally applicable (1985a) have been used in analysis to simulate indentation. The analysis cannot account for friction, face loading, or predictions.

The major factor in the choice of material properties is the behavior of ice under isotropic versus pressure-induced anisotropy in the mechanical properties. The choice between the ability to model the spatial and temporal influence of the ice and (v) the pressure in the design.

#### NUMERICAL MODEL

The research involves the development of an inelastic mechanical behavior finite element material behavior model to dominate, and to be used in the context of these contact problems. The report describes the work of Sunder (1985b).

#### Constitutive

The constitutive function describes the linear generalization of the associated Prager surface representing the material behavior depending on the characteristics of the material.

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dependent material behavior explicitly, necessitates the empirical definition of an average strainrate measure for the ice sheet. Theoretical predictions of interface pressures are not generally available. However, Ting and Shyam Sunder (1985a) have applied the (approximate) strain path method of analysis to study interface pressures during plane strain indentation. Their study showed that approximate methods of analysis cannot adequately model interface adfreeze and friction, factors that can significantly influence ice load predictions.

The major factors addressed in this paper include: (i) the choice of material model for describing the mechanical behavior of ice, i.e., plastic versus nonlinear viscoelastic, isotropic versus anisotropic and pressure-sensitive versus pressure-insensitive; (ii) the influence of natural variability in the mechanical properties of sea ice, i.e., the variability in the constants of the material models; (iii) the choice between approximate and "exact" methods of analysis and the ability of each to model interface adfreeze and friction, and spatial and temporal variability of strainrate field; (iv) the influence of grounded rubble pile or accreted ice foot; and (v) the prediction of pressure-area curves that are used in the design of structural components.

#### NUMERICAL MODELING

The research contributions to date include: (1) the development of an integrated constitutive theory for describing the mechanical behavior of sea ice, (2) the development of a finite element method of analysis for problems in which the material behavior is rate-sensitive and inelastic deformations dominate, and (3) the study of steady state sea ice indentation in the creeping mode through numerical simulations. Each of these contributions is discussed below. A comprehensive report describing the work has been written by Ting and Shyam Sunder (1985b).

##### Constitutive modeling

The constitutive theory for sea ice integrates (i) a potential function description for modeling deformations based on a nonlinear generalization of the Maxwell differential model and the associated flow rule with (ii) a rate-sensitive Drucker-Prager surface to describe ultimate failure by macrocracking representing either yielding of the material or fracture depending on the stress state. The constitutive model is characterized by its ability to:

- (a) Decompose the various recoverable (instantaneous and delayed elastic or primary creep) and irrecoverable (secondary creep and strain-softening or tertiary creep) components of strain.

- (b) Represent continuously damaging or strain-softening material behavior during ductile-to-brittle transition in compression with a linear incremental damage accumulation model.
- (c) Describe materially anisotropic mechanical behavior with a pressure-insensitive but rate-dependent potential function.
- (d) Predict first crack occurrence or nucleation with a rate-dependent limiting tensile strain criterion.
- (e) Distinguish the mechanisms of multiaxial flow as a continuum and ultimate failure by macrocracking leading to yielding of the material or fracture.

Further, the model shows strong dependency of the continuum behavior under creep and constant strainrate conditions. The model predictions compare very well with several independent sets of experimental data, particularly those for first-year sea ice. Data for the uniaxial "strength" of sea ice has been augmented with the extensive experimental data base available for pure polycrystalline ice through a normalization proposed by Weeks and Assur (1967) based on the work of Frankenstein and Garner (1967) to account for the presence of brine. The effect of temperature on the continuum behavior of ice is modelled in terms of an Arrhenius activation energy law that is considered by Mellor (1983) to be valid for temperatures below  $-10^{\circ}$  Celsius. This model is described in two papers by Ting and Shyam Sunder (1986a,b); the first focusses on the continuum behavior of sea ice while the second focusses on its yielding and fracture behavior. A companion paper at this conference relates the model to the micromechanical behavior of ice (Shyam Sunder, 1986).

#### Finite element modeling

The finite element formulation for general viscoplastic behavior (including nonlinear viscoelasticity) is based on the displacement method and is derived from a weighted equilibrium-rate equation. The use of a rate formulation allows realistic simulation of ice behavior since the spatial-temporal variability in the strainrate field can be explicitly modelled and no empirical definition of an average strain-rate measure, such as  $V/4D$  or  $V/2D$  where  $V$  is the ice sheet velocity and  $D$  is the indenter diameter, is necessary as in plasticity theory. A bi-level solution algorithm, which enables (fast) convergence in problems where inelastic deformations dominate, has been developed to solve the pseudo-force form of the nonlinear governing equations. At each time step, a successive substitution type iteration is applied to the system (global) equations of motion while a Newton-Raphson or tangent type nonlinear equation solver combined with the  $\alpha$ -

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method of numerical time integration is applied to the constitutive equations at the Gauss integration points in the finite element grid. Variable interface conditions between the ice feature and the structure can be simulated to bound the effects of interface adfreeze or friction. In particular, a "free" interface condition, which represents no interface adfreeze or friction, is simulated by an adaptive procedure that allows only normal compressive stresses to develop at the interface. The finite element formulation for isotropic behavior of ice is discussed in Chehayeb et al. (1985), while that for anisotropic material behavior is developed in Ganguly (1986). Connor and Chehayeb (1986) summarize this research.

#### Numerical simulations

Numerical simulations are performed under plane stress conditions to study the sea ice indentation problem for wide structures under steady state creep conditions. Creep is the predominant mode of deformation for artificial islands in the Arctic nearshore region during "breakout" and/or steady indentation conditions occurring during winter. Further, stresses, strains and strainrates resulting from creep are necessary to predict the nucleation, growth initiation and propagation of cracks when viscous effects influence fracture behavior. The latter scenario, which defines the transition from ductile to brittle behavior, may represent the most severe loading case since indentation pressures tend to be lower at faster ice movement rates.

The mechanical behavior of sea ice is modelled in terms of an elastic - power law creep formulation that can simulate both isotropic and anisotropic behavior. For columnar sea ice with randomly oriented c-axis in the plane of the ice sheet, material anisotropy can be described with a transversely isotropic formulation. The numerical simulations quantify the effect of this type of anisotropy on ice loads. Furthermore, the material model is pressure insensitive, i.e., the hydrostatic stress state does not influence the mechanical behavior of ice. This is unlike the plasticity-based pressure sensitive formulations of Reinicke and Ralston (1977) and Reinicke and Remer (1978) that are justified on the basis of Frederking's (1977) data on the plane strain compressive strength of columnar-grained and granular-snow ice and Jones' (1978, 1982) triaxial data for pure polycrystalline ice. Shyam Sunder et al. (1986) show that Frederking's data follows a pressure insensitive model very well and that for sea ice, which is significantly less pressure sensitive than pure ice (see Richter-Menge, Cox et al., 1985, for first-year sea ice data), a pressure insensitive formulation may be adequate in many engineering applications. In the present study material damage as a result of microcracking which leads to strain-softening and/or tertiary creep is neglected; this yields conservative predictions of ice loads.

The numerical simulations are performed by specifying a uniform far-field velocity for the ice sheet. A range of velocities are considered around a reference or base velocity. The chosen base velocity of 0.195 m/hr corresponds to the recorded maximum average velocity over a twelve-hour period just prior to "breakout" (macrocracking) for an artificial island in the Beaufort Sea. This is selected in order to predict the maximum ice loads on the structure which is assumed to occur just prior to macrocrack formation or "breakout" where ice deforms mainly in the creeping mode. Macrocracks tend to relieve the stresses built up in an ice sheet and as such reduce ice loads.

Two extreme scenarios are considered to quantify the effect of interface adfreeze and friction: in the first case the interface is considered to be infinitely strong and is simulated with a "fixed" condition that considers the interface nodes of the ice sheet and structure to be connected; while in the second case no interface adfreeze bond and friction stresses are allowed to develop using a "free" condition that permits only normal compressive stresses at the interface. The "roller" condition, which has little physical significance as an interface condition during indentation, provides an intermediate solution.

#### SUMMARY OF RESULTS

The major findings from the numerical simulation studies, discussed fully in Chehayeb et al. (1985) and Shyam Sunder et al. (1986), are summarized below:

1. Global forces can vary by a factor of up to 2.5 depending upon the magnitude of adfreeze bond strength or frictional stresses at the ice-structure interface.
2. Finite element predictions of global forces and local pressures differ from a new (approximate) modified upper bound solution by less than about 10%. This upper bound solution corresponds to the two-dimensional velocity field postulated by Ting and Shyam Sunder (1985a) and is obtained for a power law model of ice behavior. The kinematic model, obtained by superimposing a uniform flow and a doublet, resembles the flow of an infinite ice sheet past a circular indenter with the interface matching most the roller condition. The approximate formula may be expressed as given below:

$$\frac{P}{Dt} = \theta(\beta) \Gamma_p(\beta) \frac{4\pi}{\sqrt{3}} \frac{N}{N+3} \sigma_c(\epsilon_a) \quad (1)$$

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3. The ratio of pressure depending on adfreeze bond or structure in development sheet behavior

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where  $P$  is the global force,  $D$  and  $t$  are the indenter diameter and thickness respectively,  $V$  is the ice sheet velocity,  $N$  is the power law exponent (typically equal to three),  $\sigma_c(\dot{\epsilon}_a)$  is the uniaxial compressive strength of sea ice at a theoretically obtained characteristic strain rate of  $\dot{\epsilon}_a = 8V/\sqrt{3}D$ , and  $\Gamma_p$  is the theoretically obtained ratio of global pressures for the transversely isotropic and isotropic cases that is a function of  $\beta$ , the ratio of uniaxial strengths at constant strainrate transverse to and in the plane of the ice sheet (typically varying from 3 to 5), and is given as:

$$\Gamma_p = \frac{\beta}{[(4\beta^n - 1)/3]^{1/n}} \quad (2)$$

with  $n = 2N/(N+1)$ . The factor  $\theta$  is used to modify the upper bound solution, which corresponds to a plane strain condition as a result of the two-dimensional kinematic field selected, in order to be able to apply it under plane stress conditions. Based on theoretical arguments contained in Shyam Sunder et al. (1986),  $\theta$  may be expressed as:

$$\theta = 0.69 - 0.19\exp[-0.7(\beta-1)] \quad (3)$$

Table 1 provides multiplying factors that can be applied to Eq. (1) to estimate the maximum (peak) interface normal stress or "local" pressure and the effect of various interface conditions on ice loads. In boundary value problems that are significantly different from that considered here the chosen kinematic field may be inadequate. In such cases numerical simulations should be performed to predict the ice loads and, if necessary, similar approximate solutions may be derived.

3. The ratio of maximum normal interface pressure to global pressure varies approximately in the range 0.36-1.16 depending on the interface condition. It is 0.36 if the adfreeze bond strength is infinite and 1.16 if no adfreeze bond or frictional stresses are present at the ice structure interface. In the former case shear stresses can develop at the interface, while in the latter case the ice sheet has negligible stresses on the downstream side and behaves essentially as a rigid body.
4. Theoretical predictions of pressure-area curves under "breakout" and/or steady indentation conditions provide an excellent match to measured local pressures as shown in

Fig. 1. The darkly shaded areas represent field and laboratory experimental data, while the lightly shaded areas represent Sanderson's (Personal Communication, 1984) extrapolation of that data. The dark region in the extreme left is from laboratory indentation tests, the central region reflects measurements from ice breakers traveling in the Arctic, while the two smaller regions on the right correspond to average global pressures on artificial islands estimated from pressure sensor measurements in the ice sheet. The contact area is defined as the indenter area of contact for the laboratory and artificial island data. For the ice breaker data, the contact area is the local area over which the pressure measurement is made and not the form area of the ice breaker.

Figure 1 contains two solid lines representing the maximum normal interface stress on the indenter (defined here as the local pressure or indentation pressure) under free interface conditions predicted by Eq. (1) for the uniaxial power law material constants derived by Sanderson (1984). The lower solid line represents isotropic material behavior while the upper line represents an extreme level of transverse isotropy with  $\beta=5$ . The contact area is taken as  $Dt$  which is the form area (not the tributary loaded area of a structure) with  $t = 2m$  and the ice sheet velocity equals  $0.195 \text{ m/hr}$ , the value just prior to "break-out". For contact areas greater than  $10 \text{ m}^2$  where plane stress conditions exist, the two lines differ by only a factor of 1.2. It is conservative to assume a uniform or rectangular distribution of local pressures over the indenter area of contact for purposes of design. When the characteristic strainrate exceeds  $5 \times 10^{-4} \text{ s}^{-1}$ , ice is assumed to have fractured (crushed) and the uniaxial strength is capped at  $5.9 \text{ MPa}$ , leading to the flat portion of the curves on the extreme left.

The theoretical predictions made here assume knowledge of the ice sheet velocity just prior to macrocracking. At higher velocities, fracture in ice will be the key mechanism that limits the pressures.

5. The presence of a grounded rubble pile at the time of "breakout" can cause the force transmitted to the foundation by the structure to either reduce by as much as a factor of four or increase by as much as a factor of two depending upon the degree of consolidation of the rubble pile, the strength of its interface with the foundation soil, and the strength of the foundation soil. This is based on an analysis which assumes that the ice sheet confronts an indenter with an effective diameter that is equal to several times (two to three) the structural diameter. Two scenarios are considered: one in which all

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#### CONCLUSIONS

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the load is transmitted to the foundation by the structure and the other in which the load is transmitted to the foundation by both the structure and the rubble pile in proportion to the contact area of each with the foundation. The stronger is the rubble pile, its interface with soil and the soil itself, the lower is the force on the structure. This is known to be the case from experience with constructed ice packs around structures.

6. The natural variability in the material constants for an isotropic power law model of sea ice can lead to loads that vary by almost a factor of five. This variability is associated with the geographic location of the ice, its morphology, presence of inhomogeneities, etc. These uncertainties can be greater for multiyear floes. Anisotropy, as represented by the relative strengths transverse to and in the plane of the ice sheet varying between one and five, can cause global forces to increase by up to 15% and peak normal interface pressures to increase by up to 20% depending upon the interface condition.
7. Even a factor of two uncertainty in the uniform far-field ice sheet velocity just prior to macrocracking affects ice loads only by about 20-30%.
8. Tensile stresses, strains and strainrates occur almost all over the ice sheet, and may be the key to explaining fracture behavior during indentation. While biaxial compression and tension states tend to occur for stresses on the upstream and downstream sides, respectively, the levels of tensile strain are often sufficient to cause cracking even before steady state creep is reached.

#### CONCLUSIONS

This paper has identified and quantified the major factors affecting ice load prediction during indentation by developing and applying numerical models for simulating ice-structure interaction processes. The power of numerical simulations in studying such important, yet basic, problems of ice engineering has been established. The full potential of this power can be harnessed only with an in-depth knowledge of the mechanical behavior of ice and the reduction of that knowledge into constitutive models of appropriate complexity.

Further research is required to (a) predict the level of force that can be directly transmitted to the foundation by a rubble pile or constructed ice pack, (b) study the influence of boundary value problems other than "breakout" and steady indentation on ice loads, and (c) study the influence of high confining pressures, temperature gradients, and fracture in problems of ice-structure interaction.

Current research at MIT is concerned with the development of numerical models to simulate fracture processes in ice deforming both as an elastic material and as a nonlinear viscoelastic material. Experimental research is being initiated to help characterize the mechanical behavior of ice in such simulations.

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#### REFERENCES

- American Petroleum Institute (1982), Bulletin on Planning, Designing, and Constructing Fixed Offshore Structures in Ice Environments, Bul. 2N, First Edition, January.
- Bruen, F.J. and Vivatrat, V. (1984), Ice Force Prediction Based on Strain-Rate Field, Proceedings, 3rd International Symposium on Offshore Mechanics and Arctic Engineering, New Orleans, LA, 3, 275-281.
- Chehayeb, F.S., Ting, S-K., Shyam Sunder, S. and Connor, J.J. (1985), Sea Ice Indentation in the Creeping Mode, Proceedings, 17th Offshore Technology Conference, OTC 5056, May, 329-341.
- Connor, J.J. and Chehayeb, F.S. (1986), Numerical Modeling of Creep Induced Ice Structure Interaction, Proceedings, Ice Technology Conference, Massachusetts Institute of Technology, Cambridge, MA, June 10-12.
- Croasdale, K.R., Morgenstern, M.R. and Nuttall, J.B. (1977), Indentation Tests to Investigate Ice Pressures on Vertical Piers, Journal of Glaciology, 19, 81, 301-312.
- Frankenstein, G. and Garner, R. (1967), Equations for Determining the Brine Volume of Sea Ice from  $-0.5^{\circ}$  to  $-22.9^{\circ}\text{C}$ , Journal of Glaciology, 6, 48, 943-944.
- Frederking, R. (1977), Plane Strain Compressive Strength of Columnar-Grained and Granular-Snow Ice, Journal of Glaciology, 18, 80, 505-516.

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Jones, S.J.  
Proceedings  
Edmonton, 7

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Ganguly, J. (1986), Finite Element Modeling of Sea Ice Indentation in the Creeping Mode, Master of Science Thesis, Massachusetts Institute of Technology, Department of Civil Engineering, Supervised by Professor S. Shyam Sunder, February, 71p.

Jones, S.J. (1978), Triaxial Testing of Polycrystalline Ice, Proceedings, 3rd International Conference on Permafrost, Edmonton, Alberta, Canada, July, 671-674.

Jones, S.J. (1982), The Confined Compressive Strength of Polycrystalline Ice, Journal of Glaciology, 28, 98, 171-177.

Mellor, M. (1983), Mechanical Behavior of Sea Ice, U.S. Army Cold Regions Research and Engineering Laboratory, CRREL Monograph 83-1, June, 105p.

Michel, B. and Toussaint, N. (1977), Mechanisms and Theory of Indentation of Ice Plates, Journal of Glaciology, 19, 81, 285-300.

Ponter, A.R.S. et al. (1983), The Force Exerted by a Moving Ice Sheet on an Offshore Structure: Part I - the Creep Mode, Cold Regions Science and Technology, 8, 109-118.

Ralston, T.D. (1978), An Analysis of Ice Sheet Indentation, Proceedings, IAHR Ice Symposium, Lulea, Sweden, 13-31.

Reinicke, K.M. and Ralston, T.D. (1977), Plastic Limit Analysis with an Anisotropic, Parabolic Yield Function, International Journal of Rock Mechanics, Mining Sciences and Geomechanics, 14, 147-154.

Reinicke, K.M. and Remer, R. (1978), A Procedure for the Determination of Ice Forces - Illustrated for Polycrystalline Ice, Proceedings, IAHR Ice Symposium, Lulea, Sweden, August, 217-238.

Richter-Menge, J.A. et al. (1985), Triaxial Testing of First-Year Sea Ice, U.S. Army Cold Regions Research and Engineering Laboratory, Internal Research Report No. 877.

Sanderson, T.J.O. (1984), Theoretical and Measured Ice Forces on Wide Structures, Proceedings, IAHR Ice Symposium, Hamburg, August, 32p.

Shyam Sunder, S. (1986), An Integrated Constitutive Theory for the Mechanical Behavior of Sea Ice: Micromechanical Interpretation, Proceedings, Ice Technology Conference, Massachusetts Institute of Technology, Cambridge, MA, June 10-12.

Shyam Sunder, S. and Ting, S-K. (1985), Ductile to Brittle Transition in Sea Ice Under Uniaxial Loading, Proceedings, 8th International Conference on Port and Ocean Engineering Under Arctic Conditions, Narssarssuaq, Greenland, September 6-13, 656-666.

Shyam Sunder, S., Ganguly, J. and Ting, S-K. (1986), Anisotropic Sea Ice Indentation in the Creeping Mode, Proceedings, 5th International Symposium on Offshore Mechanics and Arctic Engineering, Tokyo, Japan, April 13-18, 11p.

Ting, S-K. and Shyam Sunder, S. (1985a), Sea Ice Indentation Accounting for Strain-Rate Variation, Proceedings, ASCE Speciality Conference: ARCTIC '85 - Civil Engineering in the Arctic Offshore, San Francisco, CA, March, 931-941.

Ting, S-K. and Shyam Sunder, S. (1985b), Constitutive Modeling of Sea Ice with Applications to Indentation Problems, Massachusetts Institute of Technology, Center for Scientific Excellence in Offshore Engineering, Departments of Civil Engineering and Ocean Engineering, CSEOE Report No. 3, October, 255p.

Ting, S-K. and Shyam Sunder, S. (1986a), A Rate-Sensitive Constitutive Model for the Continuum Behavior of Sea Ice, Submitted for Publication.

Ting, S-K. and Shyam Sunder, S. (1986b), A Rate-Sensitive Constitutive Model for the Yielding and Fracture Behavior of Sea Ice, Submitted for Publication.

Vivাত্র, V. and Chen, V. (1985), Ice Load Prediction with the Use of a Rate-Dependent Anisotropic Constitutive Law, Proceedings, ASCE Speciality Conference: ARCTIC '85 - Civil Engineering in the Arctic Offshore, San Francisco, CA, March, 942-952.

Vivাত্র, V., Chen, V. and Bruen, F.J. (1984), Ice Load Prediction for Arctic Nearshore Zone, Cold Regions Science and Technology, 26p.

Weeks, W. and Assur, A. (1967), The Mechanical Properties of Ice, U.S. Army Cold Regions Research and Engineering Laboratory, CRREL Report II-C3, 80p.



Table 1 Multiplying Factors for Approximate Model

Condition	Average Global Pressure	Maximum Interface Normal Stress
Roller	1.00	0.56
Fixed	1.27	0.46
Free	0.50	0.57

Note: Factor for Maximum Interface Shear Stress in Fixed Condition = 0.33

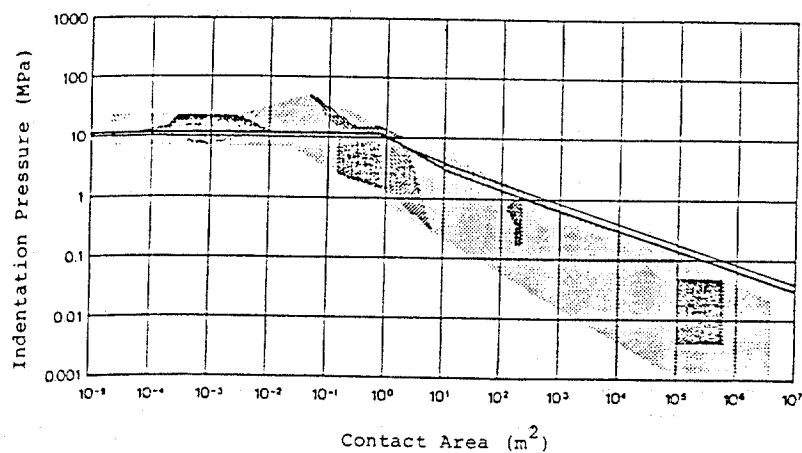


Figure 1 Pressure-Area Curve